CALCULUS BC

3.9 Applications of Derivatives

What is the relationship between position, velocity and acceleration?

$s(t)$ position $v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

Total distance =|s(b)-s(a)|

Speed = |v(t)|

Ex.1

A particle moves along a straight line for t≥0, the position of the particle is given by $s(t)=t^3-6t^2+9t$, where t is measure in seconds, and s in meters.

a) Find the velocity at any time t.
b) Find the velocity at t=2 and t=4.
c) Find all times the particle is at rest.
d) When is the particle moving in a positive direction?
e) When is the particle moving in a negative direction?
f) Draw a diagram to represent the motion of the particle.
g) Find the total distance traveled by the particle on [0,5].
h) Find the acceleration of the particle at any time t.
i) Find the acceleration of the particle at t=4.
j) Graph the position, velocity and acceleration on [0,5].
k) When is the particle speeding up?
l) When is the particle slowing down?

a) $s(t) = t^3 - 6t^2 + 9t$

$v(t) = s'(t) = 3t^2 - 12t + 9$

b) $v(2) = s'(2) = 3(2)^2 - 12(2) + 9$

$= \underline{-3 \text{ m/s}}$

$v(4) = s'(4) = \underline{9 \text{ m/s}}$

c) $v(t) = 0$

$3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$\underline{t = 1, 3}$

d) v(t) in a sign chart

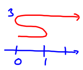
$$\begin{array}{c|c|c|c} & 0 & 0 & \\ + & & - & + \\ 0 & 1 & & 3 \end{array}$$

positive direction $(0,1) \cup (3,\infty)$

$0 < t < 1 \quad 3 < t < \infty$

e) negative direction $(1,3)$.

f)



g) total distance travelled

$s(5) - s(0)$

$|s(1) - s(0)| + |s(3) - s(1)| + |s(5) - s(3)|$

$= |4-0| + |0-4| + |20-0|$

$4 + 4 + 20$

$= \underline{28}$

The particle travels 28 m.

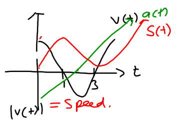
h) $a(t) = v'(t)$

$= \underline{6t - 12}$

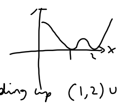
i) $a(4) = 6(4) - 12$

$= \underline{12 \text{ m/s}^2}$

j)



k)



Speeding up $(1,2) \cup (3,\infty)$

Velocity is <u>positive</u> and <u>increasing</u> so speeding up

(negative and decreasing)

l) slowing down $(0,1) \cup (2,3)$

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Ex. 2

Inflating a bicycle tire changes its radius from 11” to 13”.

a) Find the average rate of change in the area of the tire.

b) Find the instantaneous rate of change in the area of the tire at r=12.

$A = \pi r^2$

$$\text{Avg ROC} = \frac{A(13) - A(11)}{13 - 11}$$

$$= \frac{169\pi - 121\pi}{2}$$

$$= \underline{24\pi \text{ in}}$$

$$\frac{dA}{dr} = 2\pi r$$

$$= 2\pi(12)$$

$$= \underline{\underline{24\pi}} \text{ in}$$

CALCULUS BC

Ex. 3
Economics

c(x) – cost of producing x items

Marginal Cost
The cost from producing ONE extra unit – (one unit could be 1000's of an item).

$$\frac{dc}{dx} = c'(x)$$

Suppose a cost function $c(x)=x^3 - 8x^2 + 16x$, where x represents thousands of units.

a) Find the marginal cost function

b) Find c'(3), c'(4), c'(5) and comment on your answers.

a) $c(x) = x^3 - 8x^2 + 16x$

$c'(x) = 3x^2 - 16x + 16$ marginal cost f^n.

b) $c'(3) = -5$

$c'(4) = 0$

$c'(5) = 11$

Aug 9-7:53 PM